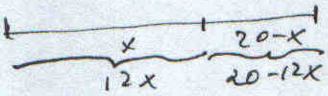
# 2009 Análisis (Parte 1)

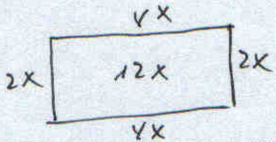
1°// Sea $f(x) = ax^3 + bx^2 + cx + d$. (0,1) es de inflexión. $f'(x) = 3ax^2+2bx+c$ $f''(x) = 6ax+2b$ $f''(0)=0$ $\Rightarrow \boxed{b=0}$

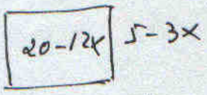
$P(0,1) \Rightarrow \boxed{1=d}$ Mínimo. $x=1$ $f'(1)=3a+2b+c=0$

$f'(2)=1 \Rightarrow 12a+4b+c=1 \Longrightarrow \begin{matrix} 3a+c=0 \\ 12a+c=1 \end{matrix}$ $9a=1$ $\boxed{a=1/9}$ $\boxed{c=-3/9=-1/3}$

2°//

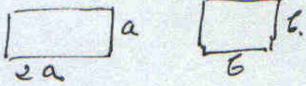






ó $A = 8x^2+(5-3x)^2$ $A' = 34x-30=0$ $x=\frac{15}{17}$ $A''=34>0 \Rightarrow$ Mínimo

$12x = \frac{180}{17}$ $20-12x=\frac{160}{17}$ ó bien.



$6a+4b=20$

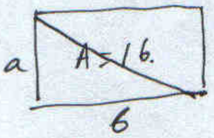
$b=\frac{20-6a}{4}$

3°// $f(x) = \sqrt{x^2-x}+x$ $[1,\infty) \longrightarrow \mathbb{R}$ AO $\Rightarrow y=2x-1/2$

Asíntota. Oblicua. $m=\lim_{x\to\infty}\frac{\sqrt{x^2-x}+x}{x}=2$

$n=\lim_{x\to\infty}\sqrt{x^2-x}+x-2x=\lim_{x\to\infty}\sqrt{x^2-x}-x=\infty-\infty=\lim_{x\to\infty}\frac{(\sqrt{x^2-x}-x)(\sqrt{x^2-x}+x)}{\sqrt{x^2-x}+x}=\lim\frac{x^2-x-x^2}{\sqrt{x^2-x}+x}=-\frac{1}{2}$

4°//



$a\cdot b=16$. $d=\sqrt{a^2+b^2}=\sqrt{a^2+(\frac{16}{a})^2}$ $d^2=a^2+(\frac{16}{a})^2=a^2+\frac{256}{a^2}$

$F' = 2a - \frac{2a\cdot 256}{a^4}=0 \Rightarrow \cancel{2}a^5-\cancel{2}\cdot 256\,a=0 \begin{cases} a=0 \\ a=\sqrt[5]{256}=2\sqrt{2} \end{cases}$

$b=8$ diagonal $=\sqrt{4+64}=\sqrt{68}$

$F''>0$

5°// $\lim_{x\to1}\left(\frac{1}{\ln x}-\frac{2}{x^2-1}\right)=\infty-\infty=\lim_{x\to1}\left(\frac{x^2-1-2\ln x}{\ln x\cdot(x^2-1)}\right)=\frac{0}{0}=$ L'hôpital $=$

$=\lim_{x\to1}\frac{2x-\frac{2}{x}}{2x\ln x+x-\frac{1}{x}}=\lim_{x\to1}\frac{\frac{2x^2-2}{x}}{\frac{2x^2\ln x+x^2-1}{x}}=\lim_{x\to1}\frac{2x^2-2}{2x^2\ln x+x^2-1}=\lim_{x\to1}\frac{4x}{4x\ln x+2x+2x}=1$

6°// $f:\mathbb{R}\longrightarrow\mathbb{R}$ a) $f(x)=\begin{cases}\frac{1}{x-1} & x<0 \\ x^2-3x-1 & x\geq 0\end{cases}$

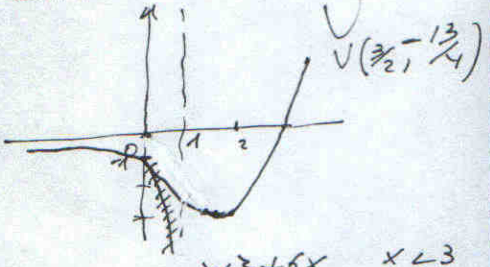
a) $\lim_{x\to0^-}\frac{1}{x-1}=-1=\lim_{x\to0^+}x^2-3x-1=-1$

$\Rightarrow$ continua en $x=0$

no es derivable en $x=0$.

$f'(x)=\begin{cases}\frac{-1}{(x-1)^2} \\ 2x-3\end{cases}$ $\lim_{x\to0^-}f'(x)=-1\neq\lim_{x\to0^+}f'(x)=-3$

b) Asíntotas. H. $y=0$ Mínimo $(3/2,-13/4)$



7°// $f:\mathbb{R}\to\mathbb{R}$ $f(x)=x^2|x-3|$ Cont. y Derivab.

$f(x)=\begin{cases}-x^2(x-3) & x<3 \\ x^2(x-3) & x\geq 3\end{cases}$

$\lim_{x\to3^-}f(x)=0=\lim_{x\to3^+}f(x)\Rightarrow$ es continua en $x=3$.

$f'(x)=\begin{cases}-3x^2+6x & x<3 \\ 3x^2-6x & x>3\end{cases}$

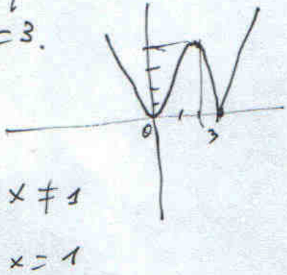
$\lim_{x\to3^-}f'(x)=-9\neq\lim_{x\to3^+}f'(x)=27-18=9$ No es derivable en $x=3$.

$y'<0$, $y'>0$, $y'<0$ | $y'>0$

dec 0 crec 2 dec 3 crec

Min(0,0) Max(2,4)

$-3x^2+6x=0 \Rightarrow \begin{cases}x=0 \\ x=2\end{cases}$



ext. relativos. Min(0,0) Max(2,4) Min(3,0)

8°// $f(x)=(0,\infty)\longrightarrow\mathbb{R}$ $f(x)=\begin{cases}\frac{x(\ln x)^2}{(x-1)^2} & x\neq 1 \\ a & x=1\end{cases}$

$\lim_{x\to1}\frac{x(\ln x)^2}{(x-1)^2}=\lim_{x\to1}\frac{(\ln x)^2+2\ln x}{2(x-1)}=\lim_{x\to1}2\ln x\cdot\frac{1}{x}+\frac{2}{x}=1\Rightarrow\boxed{a=1}$

A.H. $\lim_{x\to\infty}\frac{x(\ln x)^2}{(x-1)^2}=\lim_{x\to\infty}\frac{(\ln x)^2+2\ln x}{2(x-1)}=\lim_{x\to\infty}\frac{2\ln x\cdot\frac{1}{x}+\frac{2}{x}}{2}=\frac{0}{2}=0\Rightarrow\boxed{y=0}$

2009.

9º// $f: \mathbb{R} \longrightarrow \mathbb{R}$ $f(x) = \begin{cases} -x^2+bx+1 & x \leq 1 \\ ax^2-5x+2a & x > 1 \end{cases}$ derivable. $\Rightarrow$ continua

$\lim_{x \to 1^-} -x^2+bx+1 = -1+b+1 = b = \lim_{x \to 1^+} ax^2-5x+2a = a-5+2a = 3a-5$

$\Rightarrow b = 3a-5$ $\lim_{x \to 1^-} f'(x) = -2+b = \lim_{x \to 1^+} f'(x) = 2a-5 \Rightarrow -2+b = 2a-5$

$f'(x) = \begin{cases} -2x+b & x<1 \\ 2ax-5 & x>1 \end{cases}$

$\begin{cases} b=3a-5 \\ b=2a-3 \end{cases}$ $3a-5=2a-3 \Rightarrow \boxed{a=2}$ $\boxed{b=1}$

10º// $f: \mathbb{R} \longrightarrow \mathbb{R}$ $f(x) = ax^3+bx^2+cx+d$ $\begin{cases} (0,0),\ (2,2) \\ f'(x) = 3ax^2+2bx+c \end{cases}$ $f'(2) = 12a+4b+c = 0$ $\boxed{d=0}$ $f'(0) = \boxed{c=0}$

$2 = 8a+4b+2c$

$12a+4b=0$

$8a+4b=2$

$\Rightarrow 4a=-2$ $\boxed{a=-1/2}$ $b=-3a=\boxed{3/2=b}$

11º// $f: \mathbb{R} \longrightarrow \mathbb{R}$ $f(x) = x+e^{-x}$ $f'(x) = 1+e^{-x} = 0 \Rightarrow e^{-x} = 1$ $-x=0 \Rightarrow x=0$

$f''(x) = e^{-x} = 0$ $\cancel{\nexists}$ no tiene solución

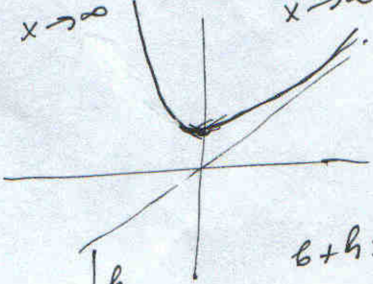
$y'<0$ dec | 0 | $y'>0$ crec.

Mimo $(0,1)$

$y''>0$ convexa.

Asintotal. Oblicua. $y = mx+n$. $m = \lim_{x \to \infty} \frac{x+e^{-x}}{x} = \lim_{x \to \infty} \frac{1-e^{-x}}{1} = 1$

$n = \lim_{x \to \infty} f(x)-mx = \lim_{x \to \infty} x+e^{-x}-x = \lim_{x \to \infty} e^{-x} = \lim \frac{1}{e^x} = 0.$ $\Rightarrow \boxed{y=x}$



12º//

h.

b

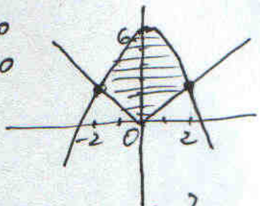
$b+h=20$ $A = \frac{b \cdot h}{2} = \frac{b \cdot (20-b)}{2} = \frac{20b}{2} - \frac{b^2}{2} = 10b - \frac{b^2}{2}$

$A' = 10 - b = 0$ $b=10.$ $A'' = -1 < 0 \Rightarrow$ Máximo.

$\Rightarrow \boxed{b=10 \quad h=10}$

③ Análisis 2009 2ª parte

1º// $f(x) = |x| \quad g(x) = 6 - x^2 \rightarrow \cap\, V(0, 6)$ $\quad f(x) = |x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$



$6 - x^2 = -x \Rightarrow x^2 - x - 6 = 0 \begin{cases} x = 3 \\ x = -2 \end{cases}$

$6 - x^2 = x \Rightarrow x^2 + x - 6 = 0 \begin{cases} x = -3 \\ x = 2 \end{cases}$

$$A = \int_{-2}^{0} (6 - x^2 + x)\,dx + \int_{0}^{2} (6 - x^2 - x)\,dx = \left(6x - \frac{x^3}{3} + \frac{x^2}{2}\right)_{-2}^{0} + \left(6x - \frac{x^3}{3} - \frac{x^2}{2}\right)_{0}^{2} = 12 - \frac{8}{3} - 2 + 12 - \frac{8}{3} - 2 =$$

$$= 24 - \frac{16}{3} - 4 = \frac{72 - 16 - 12}{3} = \boxed{\frac{44}{3}}$$

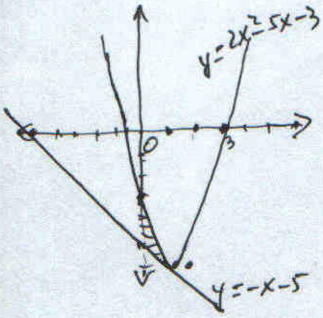
2º// $f(x) = mx^2 + nx - 3 \quad (1, -6) \quad y = -x \quad f'(x) = 2mx + n \quad f'(1) = -1 \Rightarrow 2m + n = -1$

$y + 6 = -(x - 1)$ ec.tg

$-6 = m + n - 3 \Rightarrow \begin{cases} 2m + n = -1 \\ m + n = -3 \end{cases} \quad m = 2 \Rightarrow n = -5$

$y = 2x^2 - 5x - 3 \quad \cup\, V\left(\frac{5}{4}, \frac{-49}{8}\right)$ P.corte $oy\ (y = -3)$

$ox\ (x = 3, x = -\frac{1}{2})$



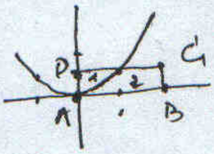
$y = -x - 5$

| x | y |
|---|---|
| 0 | -5 |
| -5 | 0 |

$$A = \int_0^1 (2x^2 - 5x - 3 + x + 5)\,dx = \int_0^1 (2x^2 - 4x + 2)\,dx =$$

$$= \left(\frac{2x^3}{3} - \frac{4x^2}{2} + 2x\right)_0^1 = \left(+\frac{2}{3} - 2 + 2\right) = \boxed{\frac{2}{3}}$$

3º// $y = \frac{1}{2}x^2$



$R_1$ y $R_2$ $\quad \begin{matrix} y = \frac{1}{2}x^2 \\ y = 1 \end{matrix} \Rightarrow x^2 = 2 \quad x = \pm\sqrt{2} \Rightarrow x = \sqrt{2}$

$$R_1 = \int_0^{\sqrt{2}} \left(1 - \frac{1}{2}x^2\right) dx = \left[x - \frac{1}{2}\frac{x^3}{3}\right]_0^{\sqrt{2}} = \sqrt{2} - \frac{1}{2}\frac{2\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}$$

$$R_2 = \int_0^{\sqrt{2}} \frac{1}{2}x^2\,dx + \int_{\sqrt{2}}^{2} dx = \left(\frac{1}{2}\frac{x^3}{3}\right)_0^{\sqrt{2}} + (x)_{\sqrt{2}}^{2} = \frac{1}{2}\frac{2\sqrt{2}}{3} + 2 - \sqrt{2} = -\frac{2}{3}\sqrt{2} + 2$$

Area total $= \boxed{2} \longrightarrow$ (área del rectángulo de base 2 y altura 1)

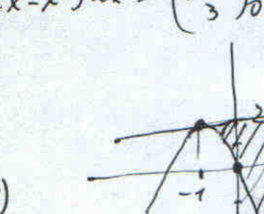
4º// $f(x) = \dfrac{x}{\sqrt{4 - 9x^4}} \quad \left[\; t = \frac{3}{2}x^2 \quad dt = 3x\,dx \;\right] = \displaystyle\int \frac{x}{\sqrt{4 - 9x^4}}\,dx = \frac{1}{3 \cdot 2} \int \frac{dt}{\sqrt{1 - (\frac{3}{2}x^2)}} =$

$$= \frac{1}{6} \int \frac{dt}{\sqrt{1 - t^2}} = \frac{1}{6} \text{arc sen} t = \frac{1}{6} \text{ar sen}\left(\frac{3}{2}x^2\right) + K$$

$F(0) = 3 \Rightarrow K = 3$

5º// $f(x) = x\,|x - 1| = \begin{cases} -x^2 + x & x < 1 \\ x^2 - x & x \geq 1 \end{cases} \quad \begin{matrix} \cap\, V(1/2, 1/4) \\ \cup\, V(1/2, -1/4) \end{matrix}$

a)



b/ $P(0,0) \quad f'(x) = 2x + 1 \quad f'(0) = 1 \Rightarrow \boxed{y = x}$

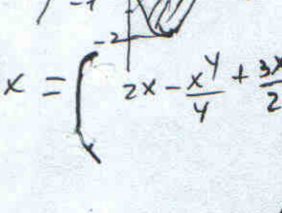
$$\text{c)}\quad A = \int_0^1 (x - (-x^2 + x))\,dx + \int_1^2 (x - (x^2 - x))\,dx = \int_0^1 x^2\,dx + \int_1^2 (2x - x^2)\,dx = \left(\frac{x^3}{3}\right)_0^1 + \left(\frac{2x^2}{2} - \frac{x^3}{3}\right)_1^2 =$$

$$= \frac{1}{3} + 4 - \frac{8}{3} - 1 + \frac{1}{3} = \boxed{1}$$

6º// $y = x^3 - 3x \quad y' = 3x^2 - 3 = 0 \quad x = \pm 1$

| $y' > 0$ | $y' < 0$ | $y' > 0$ |
|---|---|---|

-1 Max(-1,2) $\quad$ 1 Min(1,-2)

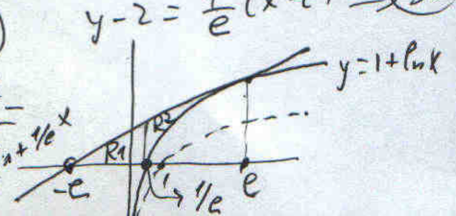


$y'(-1) = 0 \Rightarrow$ ec. tg. $y = 2$

$$A = \left[\; y = x^3 - 3x = 2 \Rightarrow x^3 - 3x - 2 = 0 \begin{cases} x = -1 \\ x = 2 \end{cases} \right] = \int_{-1}^{2} (2 - (x^3 - 3x))\,dx = \left(2x - \frac{x^4}{4} + \frac{3x^2}{2}\right)_{-1}^{2} =$$

$$= 4 - 4 + 6 - \left(-2 - \frac{1}{4} + 3/2\right) = 6 + 2 + 1/4 - 3/2 = 8 - \frac{5}{4} = \boxed{\frac{27}{4}}$$

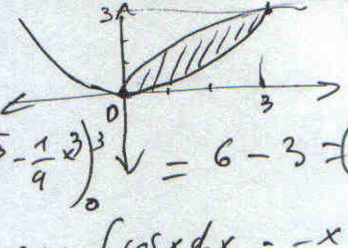
7º// $f(x) = 1 + \ln(x) \quad y = 1 + \frac{1}{e}x \quad f'(x) = \frac{1}{x} \quad f'(e) = \frac{1}{e} \quad P(e, 2) \quad y - 2 = \frac{1}{e}(x - e) \Rightarrow \boxed{\text{Si}}$



$$R = \left(e + \frac{1}{e}\right) \cdot 1/2 + R_2 \quad R_2 = \int_{1/e}^{e} \left(1 + \frac{1}{e}x - 1 - \ln x\right) dx = \left(\frac{x^2}{2e} - (x\ln x - x)\right)_{1/e}^{e} =$$

$$= \left(e + 1/e - \frac{2}{e} - \frac{1}{2e^3}\right) \quad = \frac{e}{2} - \frac{1}{2e^3} - \frac{2}{e}$$

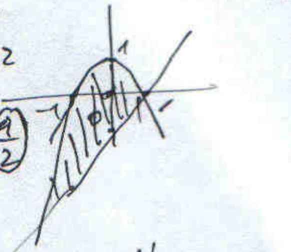
④ 2009 Análisis.



8º// $f(x)=\sqrt{3x}$ $(0,\infty)$ $g(x)=\frac{1}{3}x^2$ $\cup$ V(0,0)

$$A=\int_0^3\left(\sqrt{3x}-\frac{1}{3}x^2\right)dx=\left(\sqrt{3}\,\frac{x^{3/2}}{3/2}-\frac{1}{3}\frac{x^3}{3}\right)_0^3=\left(\frac{2\sqrt{3}}{3}\sqrt{x^3}-\frac{1}{9}x^3\right)_0^3=6-3=\enclose{circle}{3}$$

9º// a) $\int x\,\text{sen}x\,dx=\begin{bmatrix}u=x & du=dx\\ dv=\text{sen}x\,dx & v=-\cos x\end{bmatrix}=-x\cos x+\int\cos x\,dx=-x\cos x+\text{sen}x+K.$



b) $f(x)=-x^2+1$ $g(x)=x-1$ $-x^2+1=x-1$ $x^2+x-2=0\begin{cases}x=1\\x=-2\end{cases}$

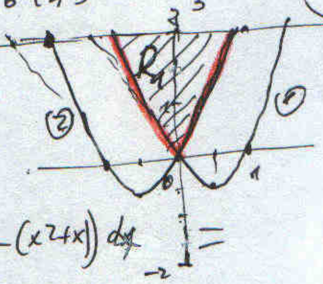
$$\int_{-2}^{1}(-x^2+1-x+1)\,dx=\left(-\frac{x^3}{3}-\frac{x^2}{2}+2x\right)_{-2}^{1}=-\frac{1}{3}-\frac{1}{2}+2-\left(\frac{8}{3}-2-4\right)=\enclose{circle}{\frac{9}{2}}$$

10º// $f(x)=\frac{2}{x}+2\ln x$ $f'(x)=-\frac{2}{x^2}+\frac{2}{x}=\frac{-2+2x}{x^2}=\frac{2x-2}{x^2}$

a/ $f'(x)=\frac{2x-2}{x^2}$ tien A.V en $x=0$ y AH = $y=0$ $\lim_{x\to0^+}f'(x)=-\infty$ $\Rightarrow$ ⑧ $\Rightarrow f'(x)$

b) $$A=\int_1^3\left[\frac{2}{x}+2\ln x-\left(\frac{2x-2}{x^2}\right)\right]dx=\int_1^3\left(\frac{\cancel{2}}{\cancel{x}}+2\ln x-\frac{\cancel{2}}{\cancel{x}}+\frac{2}{x^2}\right)dx=\left(2(x\ln x-x)+\frac{2x^{-1}}{-1}\right)_1^3=$$

$$=\left(2x\ln x-2x-\frac{2}{x}\right)_1^3=6\ln 3-6-\frac{2}{3}-(-2-2)=6\ln 3-6-\frac{2}{3}+4=\enclose{circle}{6\ln 3-\frac{8}{3}}$$



11º// $f(x)=x^2+|x|=\begin{cases}① \; x^2-x & x<0 \;\cup V(1/2,-1/4)\\ ② \; x^2+x & x\geq 0\;\cup V(-1/2,1/4)\end{cases}$ $g(x)=2$

$x^2-x=2$ $x^2-x-2=0\begin{cases}x=2\\x=-1\end{cases}$

$x^2+x=2$ $x^2+x-2=0\begin{cases}x=-2\\x=1\end{cases}$

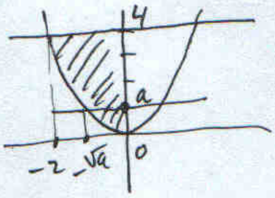
$$A=\int_{-1}^{0}\left(2-(x^2-x)\right)dx+\int_0^1\left(2-(x^2+x)\right)dx=$$

$$=\int_{-1}^{0}\underbrace{(-x^2+x+2)}_{R_1}dx+\int_0^1(-x^2-x+2)dx=\left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)_{-1}^{0}+\left(-\frac{x^3}{3}-\frac{x^2}{2}+2x\right)_0^1=$$

$$=-\frac{1}{3}-\frac{1}{2}+2-\frac{1}{3}-\frac{1}{2}+2=4-1-\frac{2}{3}=3-\frac{2}{3}=\enclose{circle}{\frac{7}{3}}$$

También se puede hacer
$R_1$ y multiplicar por 2

12º// $a<4$ $y=x^2$ $y=4$ $y=a$ Area $=\frac{28}{3}$



$$R=14/3=\int_{-2}^{0}(4-x^2)dx-\int_{-\sqrt{a}}^{0}(a-x^2)dx=\left(4x-\frac{x^3}{3}\right)_{-2}^{0}-\left(ax-\frac{x^3}{3}\right)_{-\sqrt{a}}^{0}=$$

$$=8-\frac{8}{3}-\left(2\frac{\sqrt{a^3}}{3}\right)\Longrightarrow\boxed{a^3=1}\Rightarrow\boxed{a=1}$$