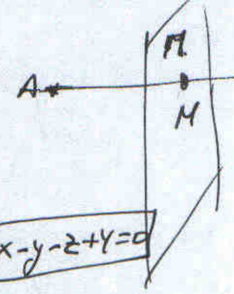
① Ejercicios de Geometría.

2010



1º// $A(1,0,2)$ $B(-1,2,4)$ $r \equiv \begin{cases} x = -2 + 2\lambda \\ y = 1 + \lambda \\ z = 1 + 3\lambda \end{cases}$

a/ $\pi \equiv$ plano que pasa por M y ⊥ $\overrightarrow{AB}$ $M = (0,1,3)$

$\overrightarrow{AB}(-2,2,2) \simeq (-1,1,1)$

$-x+y+z+d=0$ $1+3+d=0$ $d=-4$ $\pi \equiv \boxed{x-y-z+4=0}$

b/ $\pi \parallel r$ $\pi \equiv \begin{cases} x = 1 - \lambda + 2\mu \\ y = \lambda + \mu \\ z = 2 + \lambda + 3\mu \end{cases} \longrightarrow 2x+5y-3z+4=0$

2º// $A(1,1,1)$ $B(0,-2,2)$ $C(-1,0,2)$ $D(2,-1,2)$ $\overrightarrow{AB}$ $\overrightarrow{AC}$ $\overrightarrow{AD}$

a) $\overrightarrow{AB}(-1,-3,1)$ $\overrightarrow{AC}(-2,-1,1)$ $\overrightarrow{AD}(1,-2,1)$

$\text{Volumen} = \frac{1}{6}\begin{vmatrix} -1 & -3 & 1 \\ -2 & -1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \frac{1}{6}(1-3+4+1-6-2) = +\frac{5}{6}$

b) $\pi \begin{cases} x = 1 - \lambda - 2\mu \\ y = 1 - 3\lambda - \mu \\ z = 1 + \lambda + \mu \end{cases}$ $\overrightarrow{AB} \wedge \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -1 & -3 & 1 \\ -2 & -1 & 1 \end{vmatrix} = -3i-2j+k-6k+j+i = -2i-j-5k \rightarrow (-2,-1,-5)$

$r \equiv \begin{cases} x = 2 + 2\lambda \\ y = -1 + \lambda \\ z = 2 + 5\lambda \end{cases}$

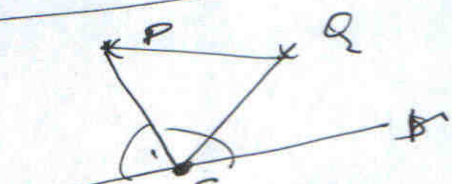
3º// a/ $r \equiv \begin{cases} x = 1 + \lambda \\ y = \lambda \\ z = 1 - \lambda \end{cases}$ $s \equiv \begin{cases} x = 1 - 2\lambda' \\ y = 1 - \lambda' \\ z = \lambda' \end{cases}$ $\begin{matrix} 1+\lambda = 1-2\lambda' \\ \lambda = 1-\lambda' \\ 1-\lambda = \lambda' \end{matrix} \Rightarrow \begin{matrix} \lambda' = -1 \\ \lambda = 2 \end{matrix}$ $P(3,2,-1)$

b/ ángulo (r,s) $\vec{u}(1,1,-1)$ $\vec{v}(-2,-1,1)$

$|u \cdot v| = |u||v| \cos\alpha$ $\cos\alpha = \frac{|-4|}{\sqrt{3}\sqrt{6}} = \frac{4}{\sqrt{18}} = \frac{4}{3\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$

$\alpha = 19'47^\circ$

c) $\pi \begin{cases} x = 1 + \lambda - 2\mu \\ y = 0 + \lambda - \mu \\ z = 1 - \lambda + \mu \end{cases}$



4º// $P(2,0,0)$ $Q(-1,12,4)$ $r \equiv \begin{cases} x = \frac{33}{4} - \frac{3}{4}\lambda \\ y = 0 \\ z = \lambda \end{cases}$

a/ No $\boxed{\overrightarrow{PS}\left(\frac{25}{4} - \frac{3}{4}\lambda, 0, \lambda\right) \quad \overrightarrow{PS} \cdot \vec{u} = 0 \quad \left(\frac{25}{4} - \frac{3}{4}\lambda, 0, \lambda\right)\left(-\frac{3}{4}, 0, 1\right) = 0}$

$r \equiv \begin{cases} x = \lambda \\ y = 0 \\ z = 11 - \frac{4}{3}\lambda \end{cases}$

$S(\lambda, 0, 11 - \frac{4}{3}\lambda)$ $\overrightarrow{PS}(\lambda - 2, 0, 11 - \frac{4}{3}\lambda)$

$(\lambda - 2, 0, 11 - \frac{4}{3}\lambda)(-3,0,4) = 0 \Longrightarrow -3\lambda + 6 + 44 - \frac{16}{3}\lambda = 0$

$\frac{25}{3}\lambda = 50$ $\lambda = \frac{150}{25} = 6$ $S(6,0,3)$

b/ PQR $PQ \cdot PS = 0$ $PQ \cdot QS = 0$ $PS \cdot QS = 0$

$\overrightarrow{PQ}(-3,12,4)$ $PS(4,0,3)$

$\hat{P} \rightarrow$ NO

$\hat{Q} \rightarrow$ NO

$\hat{S} \rightarrow$ NO.

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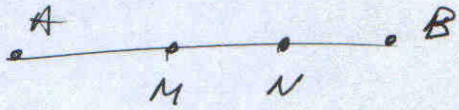
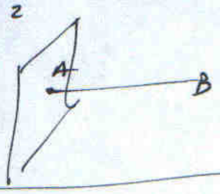
5º// $A(1,2,1)$ $B(-1,0,3)$



a) $\vec{AB}(-2,-2,2)$ $3\vec{AM}=\vec{AB}$ $3(m_1-1, m_2-2, m_3-1)=(-2,-2,2) \Rightarrow \boxed{m_1=1/3 \quad m_2=4/3 \quad m_3=5/3}$

$\frac{\frac{1}{3}-1}{2}=n_1 \Rightarrow n_1=-2/6$ $\quad \frac{4/3+0}{2}=n_2 \Rightarrow n_2=\frac{4}{6}$ $\quad \frac{5/3+3}{2}=n_3 \Rightarrow n_3=14/6$

b)

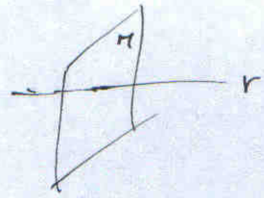


$-2x-2y+2z+d=0$

$-2-4+2+d=0 \quad d=4 \Rightarrow \pi \equiv \boxed{x+y-z=-2}$

6º// $\pi \equiv 2x-y+nz=0$ $\quad r \equiv \begin{cases} x=1+\lambda m \\ y=4\lambda \\ z=1+2\lambda \end{cases}$

a) $r \perp \pi$.



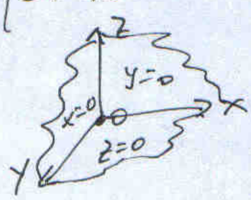
$(2,-1,n)=k(m,4,2) \Rightarrow k=-1/4$

$(2,-1,n)=-1/4(m,4,2) \Rightarrow \boxed{m=-8 \quad n=-1/2}$

b) $(2,-1,n)\cdot(m,4,2)=0 \Rightarrow \begin{cases} m-4+2n=0. \\ 2+n=0. \end{cases}$ $\quad \boxed{m=4 \quad n=-2}$

$(1,0,1)\in\pi \Rightarrow 2x-y+nz=0 \longrightarrow$

7º// $\pi \equiv 6x+3y+2z=6$



$A \equiv \pi \cap OX$ $\quad (1,0,0)$

$B \equiv \pi \cap OY$ $\quad (0,2,0)$

$C \equiv \pi \cap OZ$ $\quad (0,0,3)$

eje $\equiv OX$ $\begin{cases} y=0 \\ z=0 \end{cases}$

eje $\equiv OY$ $\begin{cases} x=0 \\ z=0 \end{cases}$

eje $\equiv OZ$ $\begin{cases} x=0 \\ y=0 \end{cases}$

$\vec{AB}(-1,2,0)$ $\quad \vec{AC}(-1,0,3)$ $\quad AB \wedge AC = \begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 6i+3j+2k$

$\Delta = 1/2\sqrt{36+9+4} = \underline{7/2}$

8º// $A(1,1,1)$ $B(-1,2,0)$ $C(2,1,2)$ $D(t,-2,2)$ $\quad$ p.d. $\equiv \vec{AB}(-2,1,-1)$ $\quad \vec{AC}(1,0,1)$ $\quad \vec{AD}(t-1,-3,1)$

a) $\begin{vmatrix} -2 & 1 & -1 \\ 1 & 0 & 1 \\ t-1 & -3 & 1 \end{vmatrix} = 0 = t-1+3-1-6 \Rightarrow t=5$

$-2x+y-z+d=0$ $\quad -4+1-2+d=0 \quad d=5$

b) $\underline{\pi \equiv -2x+y-z=-5}$ $\quad s\begin{cases} x=1-5\lambda \\ y=-2+3\lambda \\ z=2+2\lambda. \end{cases}$ $\quad \pi \equiv \begin{cases} x=1-5\lambda+2\mu \\ y=-2+3\lambda+1\mu \\ z=2+2\lambda-2\mu \end{cases}$

9º// $r \equiv \begin{cases} x-2y+11=0 \\ 2y+z-19=0 \end{cases}$

$r \equiv \begin{cases} x=-11+2\lambda \\ y=\lambda \\ z=19-2\lambda \end{cases}$

$\begin{vmatrix} x-1 & -5 & 2 \\ y+2 & 3 & 1 \\ z-2 & 2 & -2 \end{vmatrix} = -6x+6-5z+10+4y+8-6z+12-2x+2-10y-20 = \boxed{-8x-6y-11z+18=0}$

3 planos paralelos.

10º// $\pi_1 \; \pi_2 \; \pi_3$ $\begin{cases} x+y=1 \\ ay+z=0 \\ x+(1+a)y+az=a+1 \end{cases}$

a) $A=\begin{pmatrix} 1 & 1 & 0 \\ 0 & a & 1 \\ 1 & 1+a & a \end{pmatrix}$ $\quad A^*=\begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & a & 1 & 0 \\ 1 & 1+a & a & 1+a \end{pmatrix}$

a) $|A| = a^2 - a = a(a-1)=0$ $\quad a\neq 0$ y $a\neq 1$ $\quad r(A)=3$

Si $a=0$ $\quad r(A)=2=r(A^*)$

Si $a=1$ $\quad r(A)=2$ $\quad r(A^*)=3$ $\quad$ S.I. $\Rightarrow \boxed{a=1}$

Si $a=0$ $\quad$ SCI (1 g.l.) $\quad$ 2 coincidents y el 3º los corta en una recta.

b)

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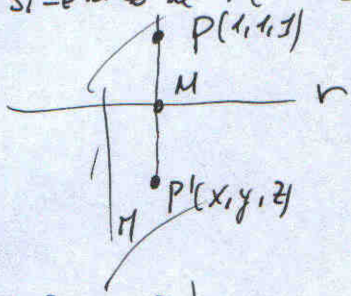
11º// Simétrico de $P(1,1,1)$ respecto de $r \equiv \begin{cases} x = 1 + 2\lambda \\ y = 3\lambda \\ z = -1 - \lambda \end{cases}$



$\pi \equiv 2x + 3y - z + d = 0 \qquad 2 + 3 - 1 + d = 0 \qquad d = -4$

$\pi \equiv 2x + 3y - z - 4 = 0 \ \cap r$

$2(1+2\lambda) + 3(3\lambda) + 1 + \lambda - 4 = 0 \qquad 2 + 4\lambda + 9\lambda + \lambda - 3 = 0$

$14\lambda = 1 \qquad \lambda = 1/14 \qquad M\left(1 + \frac{2}{14}, \frac{3}{14}, -1 - \frac{1}{14}\right)$

$M\left(\frac{16}{14}, \frac{3}{14}, \frac{-15}{14}\right)$

$\frac{1+x}{2} = \frac{16}{14} = \frac{8}{7} \Rightarrow 7 + 7x = 16 \qquad 7x = 9$

$\frac{1+y}{2} = \frac{3}{14} \Rightarrow 14 + 14y = 6 \qquad 14y = -8$

$\frac{1+z}{2} = \frac{-15}{14} \Rightarrow 14 + 14z = -30 \qquad 14z = -44$

$\boxed{x = \frac{9}{7} \quad y = -\frac{4}{7} \quad z = \frac{-22}{7}}$

12º// $A(2,\lambda,\lambda) \quad B(-\lambda,2,0) \quad C(0,\lambda,\lambda-1)$

a) $\begin{vmatrix} 2 & \lambda & \lambda \\ -\lambda & 2 & 0 \\ 0 & \lambda & \lambda-1 \end{vmatrix} = 0 = 4\lambda - 4 - \lambda^3 + \lambda^3 - \lambda^2 = -\lambda^2 + 4\lambda - 4 \Rightarrow \lambda^2 - 4\lambda + 4 = 0$

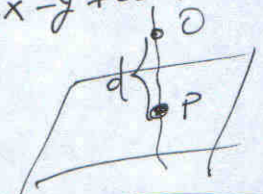
$\lambda = \frac{4 \pm \sqrt{16-16}}{2} = 2 \Longrightarrow \boxed{\lambda = 2}$

b) $\lambda = 1 \qquad A(2,1,1) \quad B(-1,2,0) \quad C(0,1,0) \qquad \vec{AB}(-3,1,-1) \quad \vec{AC}(-2,0,-1)$

$\boxed{\pi \equiv \begin{cases} x = 2 - 3\lambda - 2\mu \\ y = 1 + \lambda \\ z = 1 - \lambda - \mu \end{cases}}$ distancia $(0,0,0) \cdot \pi$

$\begin{vmatrix} x-2 & -3 & -2 \\ y-1 & 1 & 0 \\ z-1 & -1 & -1 \end{vmatrix} = -x + 2 + 2y - 2 + 2z - 2 - 3y + 3 = -x - y + 2z + 1 = 0$

$\pi \equiv \boxed{x + y - 2z = 1}$



$r \equiv \begin{cases} x = \lambda \\ y = \lambda \\ z = -2\lambda \end{cases} \qquad r \cap \pi = \lambda + \lambda + 4\lambda = 1 \qquad \lambda = \frac{1}{6}$

$P(1/6, 1/6, -1/3) \qquad d = \sqrt{(1/6)^2 + 1/6^2 + \left(\frac{2}{6}\right)^2} =$

$d = \sqrt{\frac{6}{36}} = \sqrt{\frac{1}{6}}$