① 2012

1º// $f: (0,\infty) \to \mathbb{R}$ $\quad f(x) = \frac{1}{x} + \ln x$

a) $y' = -\frac{1}{x^2} + \frac{1}{x} = 0 \Rightarrow -1 + x = 0 \Rightarrow x = 1$

$\frac{1}{e}$ — $y'<0$ — 1 — $y'>0$ — e

Min(1,1)

$f(1/e) = e + \ln e^{-1} = e + -1 \simeq 1'71$

$f(e) = \frac{1}{e} + \ln e = \frac{1}{e} + 1 \simeq 1'36$

Máx. absoluto $(1/e, e-1)$ Mínimo absol = mín. relat = $(1,1)$

b) $y - y_0 = y'(x_0)(x - x_0)$ en $\left(e, \frac{1}{e}+1\right)$ $\quad y'(e) = -\frac{1}{e^2} + \frac{1}{e}$

$y - \frac{1}{e} - 1 = \left(-\frac{1}{e^2} + \frac{1}{e}\right)(x - e)$ ec. tg.

2º// $f(x) = \frac{2x^2}{(x+1)(x-2)}$ $\quad$ a) Asíntotas: Verticales $x = -1$ $\quad x = 2$; Horizontales $y = 2$

b) Crec. Dec. $y' = \frac{4x(x+1)(x-2) - (2x-1)2x^2}{(x^2-x-2)^2} = \frac{\cancel{4x^3} - 4x^2 - 8x - \cancel{4x^3} + 2x^2}{(x^2-x-2)^2} = \frac{-2x^2 - 8x}{(x^2-x-2)^2} = 0$

$-2x(x+4) = 0 \begin{cases} x = 0 \\ x = -4 \end{cases}$

$y'<0$ dec -4 $y'>0$ crec -1 $y'>0$ crec 0 $y'<0$ dec 2 $y'<0$ dec

c) $2 = \frac{2x^2}{x^2-x-2} \Rightarrow 2x^2 = 2x^2 - 2x - 4 \Rightarrow 2x = -4 \quad x = -2$ P. corte $(-2, 2)$

3º// $f(x): [1, e] \to \mathbb{R}$ $\quad f(x) = x^2 - 8\ln x$

a) Crec. Decrec. $y' = 2x - \frac{8}{x} = 0$ $\quad 2x^2 - 8 = 0 \quad x^2 = 4 \quad x = 2$

b) 1 dec $y'<0$ 2 crec $y'>0$

Min$(2, -1'54)$

$f(1) = 1$ $\quad f(e) = e^2 - 8\ln e = e^2 - 8 \simeq -0'61.$

Máx absoluto $(1,1)$

Min absoluto = Min relativo = $(2, -1'54)$

c) Conc. Convex. $y'' = 2 + \frac{8}{x^2} = 0$ $\quad 2x^2 + 8 = 0$ $\quad \cancel{x^2 = -4}$

1 $y''>0$ e

convexa.

4º// $f(x) = e^x(x^2 - x + 1)$ $\quad$ a) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \frac{x^2 - x + 1}{\frac{1}{e^x}} = \frac{\infty}{\infty} \Rightarrow$ L'hôpital $=$

$= \lim\limits_{x \to -\infty} \frac{2x-1}{-e^{-x}} = \lim\limits_{x \to -\infty} \frac{2}{e^{-x}} = \frac{2}{\infty} = 0$

$\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} e^x(x^2 - x + 1) = \lim\limits_{x \to \infty} \frac{2}{e^{-x}} = \infty$

b) ext. relativos

$y' = e^x(x^2 - x + 1) + (2x-1)e^x = e^x(x^2 - x + 1 + 2x - 1) = e^x(x^2 + x) = 0 \begin{cases} \cancel{e^x = 0} \\ x^2 + x = 0 \end{cases}$

$x(x+1) = 0 \begin{cases} x = 0 \\ x = -1 \end{cases}$

$y'>0$ crec -1 $y'<0$ dec 0 $y'>0$ crec

Max$\left(-1, \frac{3}{e}\right)$ Min$(0,1)$

c) P. inflexión $y'' = e^x(x^2 - x + 1) + (2x-1)e^x + e^x(2x-1) + 2e^x = e^x(x^2 - x + 1 + 2x - 1 + 2x - 1 + 2) = e^x(x^2 + 3x + 1) = 0$

$\begin{cases} \cancel{e^x = 0} \\ x^2 + 3x + 1 = 0 \end{cases}$ $\quad x = \frac{-3 \pm \sqrt{5}}{2} = -0'38, \; -2'61$

$y''>0$ convexa $\frac{-3-\sqrt{5}}{2}$ $y''<0$ concava $\frac{-3+\sqrt{5}}{2}$ $y''>0$ convexa.

$\Rightarrow$ P. inflexión en $x = \frac{-3-\sqrt{5}}{2}$ $\quad x = \frac{-3+\sqrt{5}}{2}$

② 2012

5º// $f(x)=\begin{cases} x+k & x\le 0 \\ \dfrac{e^{x^2}-1}{x^2} & x>0 \end{cases}$ a) $f(x)$ continua. $\Rightarrow \lim\limits_{x\to 0^-} x+k = \lim\limits_{x\to 0^+} \dfrac{e^{x^2}-1}{x^2}$

$\lim\limits_{x\to 0} x+k = k$; $\lim\limits_{x\to 0} \dfrac{e^{x^2}-1}{x^2} = \dfrac{0}{0} \Rightarrow$ L'hôpital $= \lim\limits_{x\to 0} \dfrac{2xe^{x^2}}{2x} = \lim\limits_{x\to 0} \dfrac{2e^{x^2}+4x^2e^{x^2}}{2}$

$= \lim\limits_{x\to 0} \dfrac{2+0}{2} = 1 \quad \Longrightarrow \boxed{K=1}$

b) $y-y_0 = y'(x_0)(x-x_0) \quad (1,\ e-1) \quad y' = \dfrac{2xe^{x^2}\cdot x^2 - 2x(e^{x^2}-1)}{x^4}$

$y'(1) = \dfrac{2e-2(e-1)}{1} = 2e-2e+2 = 2$

ec. tg. $\boxed{y-e+1 = 2(x-1)}$

6º// $f(x) = \dfrac{e^{-x}}{1-x}$ a) Asintotas A.V $\Rightarrow x=1$ A. Oblicua. $y=mx+n$.

A.H $\Rightarrow y=k \quad \lim\limits_{x\to\pm\infty} f(x) = k$

A. H. $\lim\limits_{x\to\infty} \dfrac{e^{-x}}{1-x} = 0 \Rightarrow y=0$

b) ext. relativos $y' = \dfrac{-e^{-x}(1-x)+e^{-x}}{(1-x)^2} = 0 \Rightarrow -e^{-x}(1-x-1)=0 \begin{cases} \cancel{e^{-x}=0} \\ x=0 \end{cases}$

$y'<0$ dec. | 0 | $y'>0$ crec | 1 | $y'>0$ crec.

Min(0,1)

7º// $f(x) = e^x(x-2)$ a) Asintotas $\lim\limits_{x\to\infty} f(x) = \infty \quad \lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} e^x(x-2) =$

L'hôpital $= \lim\limits_{x\to-\infty} \dfrac{x-2}{\frac{1}{e^x}} = \lim\limits_{x\to-\infty} \dfrac{x-2}{e^{-x}} = \lim\limits_{x\to-\infty} \dfrac{1}{-e^{-x}} = 0$

b) ext. relativos. $y' = e^x(x-2)+e^x = e^x(x-2+1) = e^x(x-1)=0 \begin{cases} \cancel{e^x=0} \\ x=1 \end{cases}$

$y'<0$ dec. | 1 | $y'>0$ crec.

Min(1, −e)

c) P. inflexión. $y'' = e^x(x-1)+e^x = e^x(x-1+1)=0 \Rightarrow \begin{cases} \cancel{e^x=0} \\ x=0 \end{cases}$

$y''<0$ concava | 0 | $y''>0$ convexa.

P. inflex (0, −2)

8º// $\lim\limits_{x\to 0} \dfrac{a\cdot \operatorname{sen} x - xe^x}{x^2} =$ finito, $=$ L'hôpital $= \lim\limits_{x\to 0} \dfrac{a\cos x - e^x - xe^x}{2x} = \dfrac{a-1}{0} \Rightarrow a=1$

$= \lim\limits_{x\to 0} \dfrac{-a\operatorname{sen} x - e^x - e^x - xe^x}{2} = \dfrac{-2}{2} = -1$

10º// $f(x) = \ln(x^2+3x+3) - x$ a) crec, dec. ext. relat. $y' = \dfrac{2x+3}{x^2+3x+3} - 1 =$

$= \dfrac{2x+3-x^2-3x-3}{x^2+3x+3} = 0 \Rightarrow -x^2-x=0 \quad x(x+1)=0 \begin{cases} x=0 \\ x=-1 \end{cases}$

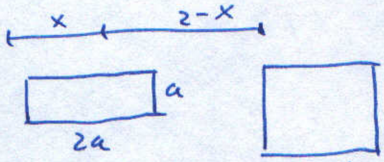
$y'<0$ dec. | −1 | $y'>0$ crec | 0 | $y'<0$ dec.

Min(−1, 1) Max(0, ln3)

$y'(-2) = -2$

b) ec. normal. (−2, 2) $y-2 = -\dfrac{1}{y'(-2)}(x+2) \Rightarrow y-2 = \dfrac{1}{2}(x+2)$

9º//



$6a = x \Rightarrow a = \frac{x}{6}$ $\quad \frac{2-x}{4}$

$A = \frac{x}{6} \cdot \frac{x}{3} + \left(\frac{2-x}{4}\right)^2 = \frac{x^2}{18} + \frac{x^2-4x+4}{16}$

$A' = \frac{2x}{18} + \frac{2x-4}{16} = 0 \Rightarrow 32x + 36x = 72 \quad x = \frac{18}{17}$

$2 - x = 2 - \frac{18}{17} = \frac{16}{17}$

10º//

$f(x) = \begin{cases} 1 + \frac{a}{x-2} & x < 1 \\ a + \frac{b}{\sqrt{x}} & x \geq 1 \end{cases}$

$f(x)$ derivable $\Rightarrow f(x)$ es continua en 1 y derivable en 1.

*Por ser continua en $x=1$

$\Rightarrow \lim_{x \to 1^-} 1 + \frac{a}{x-2} = \lim_{x \to 1^+} \frac{a+b}{\sqrt{x}} \Rightarrow$

$1 + \frac{a}{-1} = 1 - a = a + b \Rightarrow 2a + b = 1$

*Por ser derivable en $x = 1$

$f'(x) = \begin{cases} -\frac{a}{(x-2)^2} & x < 1 \\ -\frac{\frac{1}{2\sqrt{x}} b}{x} & x \geq 1 \end{cases}$

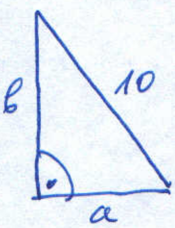
$\lim_{x \to 1} \frac{-a}{(x-2)^2} = \lim_{x \to 1} \frac{-\frac{b}{2\sqrt{x}}}{x} \Rightarrow$

$-a = -\frac{b}{2} \Rightarrow a = b/2$

$2b = 1 \Rightarrow a = b/2 = 1/4$

$\boxed{b = 1/2 \quad a = 1/4}$

12º//



$a^2 + b^2 = 10^2 \quad A = \frac{a \cdot b}{2} = \frac{a \cdot \sqrt{100 - a^2}}{2}$

$F = \frac{a^2(100-a^2)}{4} \quad F' = \frac{-4a^3 + 200a}{4} = -a^3 + 50a = 0$

$a(-a^2 + 50) = 0 \begin{cases} a = 0 \\ a^2 = 50 \Rightarrow a = \sqrt{50} \end{cases}$

$F'' = \frac{-12a^2 + 200}{4} = -3a^2 + 50 \quad F''(\sqrt{50}) < 0 \Rightarrow$ Máximo.

$\boxed{a = \sqrt{50} \quad b = \sqrt{100 - 50} = \sqrt{50}}$